

G.C.E.(O.L.) Support Seminar - 2014

Mathematics I

Two Hours

Part A

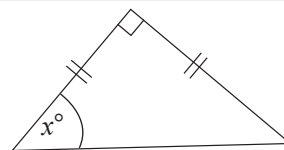
Answer **all** questions on this question paper itself.

1. Simplify : $1.2 + 0.35$

2. Find the balance when a Rs. 100 note is tendered to buy a book of value Rs. 78.

3. Simplify : $\frac{3}{5} - \frac{2}{5}$

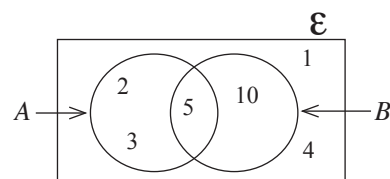
4. Find the value of x in the figure.



5. Solve : $5 + x = 9$

6. Express in grammes : 0.12 kg

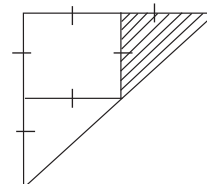
7. Find $n(A')$ based on the information in the Venn diagram.



8. Express the perimeter of a regular pentagon of side length a in terms of a .

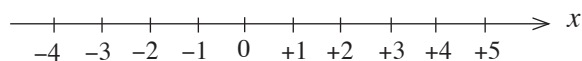
9. What is the size of the class interval 0 – 5?

10. Write down the area of the shaded region in the figure as a fraction of the area of the whole figure.



11. Simplify : $110_{\text{two}} + 11_{\text{two}}$

12. Represent the **positive integral** solutions of the inequality $2x < 6$ on the given number line.



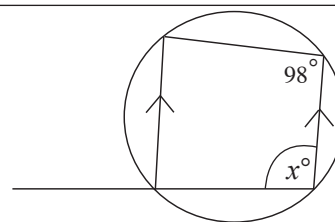
13. $12 = 2 \times 2 \times 3$

$15 = 3 \times 5$

$24 = 2 \times 2 \times 2 \times 3$

Find the, (i) Highest Common Factor (ii) Least Common Multiple of 12, 15 and 24.

14. Find the value of x based on the information in the figure.



15. Simplify : $\frac{2x}{x-1} + \frac{2}{1-x}$

16. What is the assessed annual value of a house which pays Rs. 500 a quarter as assessment tax to a local government authority that charges rates of 5% annually?

17. Make x the subject of the formula $x = \frac{a+x}{t}$.

18. Write down the coordinates of the intersection point of the two straight lines given by the equations $y = 2x - 1$ and $x = 3$.

19. Obtain the area of an equilateral triangle of side length $2a$ in terms of a . (Give the answer as a surd.)

20. Solve : $\log_2 x = 5 - \log_3 81$

21. Simplify : $3\sqrt{2} + \sqrt{8}$

22. Consider the experiment of a card being selected at random from a set of ten identical cards on which the numbers 1 to 10 (one number on each card) are written.

A – Obtaining a prime number less than 10.

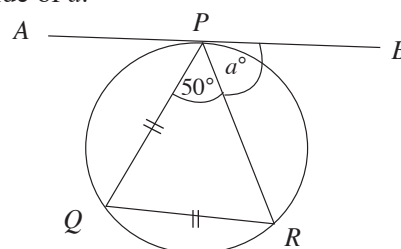
B – Obtaining an even number less than 10.

C – Obtaining a perfect square number less than 10.

By considering the above events, mark \checkmark in front of the correct statements and \times in front of the incorrect statements.

| | |
|-------------------------------------|--|
| A and B are mutually exclusive. | |
| B and C are mutually exclusive. | |
| A and C are mutually exclusive. | |

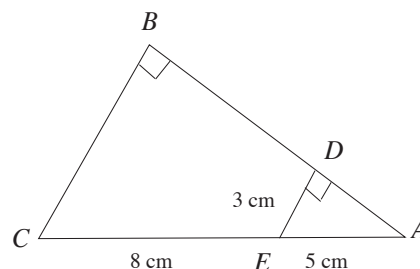
23. AB is the tangent drawn to the circle at point P . Find the value of a .



24. Based on the information in the figure,

(i) write down a ratio equivalent to the ratio $AD : DB$ in terms of the sides of the triangles.

(ii) calculate the length of AB .

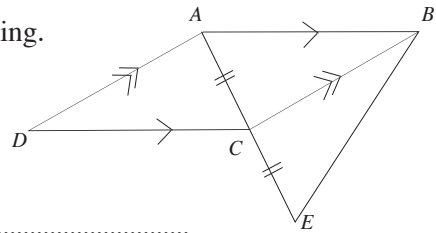


25. The magnitude of an interior angle of a certain regular polygon is three times the magnitude of an exterior angle. Find,
(i) the magnitude of an exterior angle (ii) the number of sides of this polygon.

26. Three machines require 6 hours to harvest the paddy in a certain paddy field. How many such machines are required to harvest the paddy in a field which is twice as large as the initial field, within 4 hours?

27. For the distribution of numbers 3, 12, 8, 15, 20, 12, 4, 22, 10, 24, 19, find,
(i) the median (ii) the inter-quartile range.

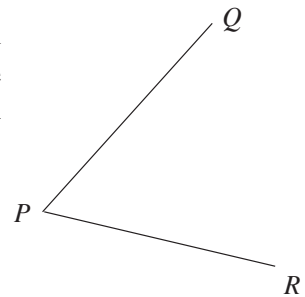
28. $ABCD$ is a parallelogram. Fill in the blanks of the following.



Area of $\triangle CBE$ = Area of Δ
(.....
.....)

\therefore Area of parallelogram $ABCD$ = Area of Δ

29. PQ and PR are two boundaries of a certain plot of land. It is required to grow a plant 5 m from the corner P and at an equal distance from the given boundaries. Using the knowledge on loci, illustrate on the given sketch how the required location is marked.



30. A certain question paper consists of 50 questions which require short answers. Each correct answer is awarded 1 mark, while 1 mark is deducted for each wrong answer. If a question is not answered, no marks are awarded or deducted. If a student who answered 40 questions obtained only 28 marks, how many questions did he answer correctly?

Part B

Answer **all** questions on this question paper itself. Each question carries 10 marks.

1. (a) Simplify : $26 \div \left(2\frac{1}{3} - \frac{1}{6}\right)$

(b) At a school variety show, $\frac{1}{3}$ of the students in the school sang, $\frac{1}{4}$ of the students danced while $\frac{4}{5}$ of the remaining students were involved in a drama. Each of these students participated in only one item.

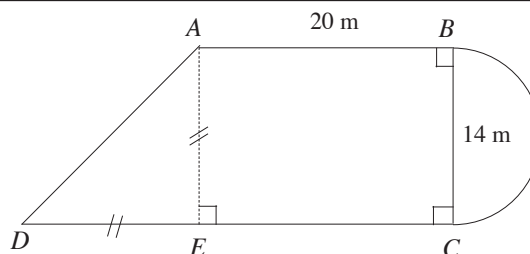
(i) What fraction of the total number of students in the school is the number of students who either sang or danced?

(ii) What fraction of the total number of students is the number that was involved in the drama?

(iii) What fraction of the total number of students is the number that **did not participate** in any of the above three items?

(iv) If the number of students who did not participate in any of the above three items is 20, how many students danced?

2. The figure illustrates a lawn $ABCD$ and a semi-circular pond adjoining the boundary BC of the lawn. In the figure $AE = DE$.



(i) What is the length of the boundary DC ?

(ii) Find the area of the lawn $ABCD$.

(iii) Find the perimeter of the pond.

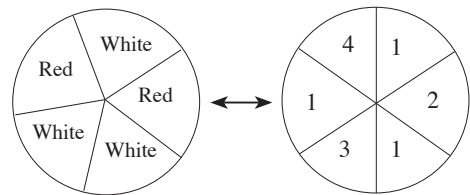
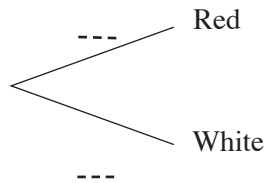
(iv) Find the area of the pond.

(v) It is proposed to create a rectangular shaped lawn with the same area, by keeping the boundary BC as before and changing the other three boundaries. Draw a sketch of the shape that is obtained with this change, and mark the relevant measurements on it.

3. (a) Chandimal, Ganesh and Razeek started a business together. At the end of the year, the profit from the business was shared among the three such that the ratio of the amount Chandimal received to the amount Ganesh received was 3 : 2 and the ratio of the amount Ganesh received to the amount Razeek received was 4 : 5.
- Write in the simplest form, the ratio according to which the profit was shared among the three.
 - If Chandimal received Rs. 20 000 more than Ganesh did, what is the total profit from the business?
 - Describe the simplest method that can be used to share the total profit equally among the three, if it has already been distributed in the above manner.
- (b) A loss of 20% is incurred by the manufacturer when a dress is sold for Rs. 1280.
- What is the production cost of the dress?
 - At what price should the dress be sold to make a profit of 20%?

4. (a) The figure illustrates "wheels of fortune" used to draw the winning ticket of a lottery. Each of the two wheels is rotated once independently of each other and released.

- (i) Using the information given in the first wheel write the relevant probabilities on the two branches of the tree diagram given below.

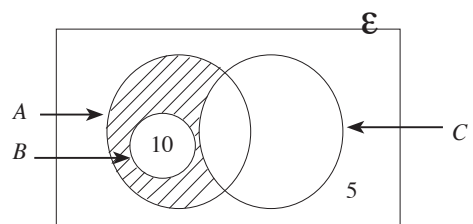


- What is the probability of obtaining a perfect square number from the second wheel?
- Extend the tree diagram to include the cases of obtaining / not obtaining a perfect square number from the second wheel, and mark the relevant probabilities on it.
- Find the probability of obtaining a perfect square number with the colour white.

- (b) In the given Venn diagram,

$$\begin{aligned} n(\mathcal{E}) &= 24 \\ n(A') &= 8 \text{ and} \\ n(C) &= 7. \end{aligned}$$

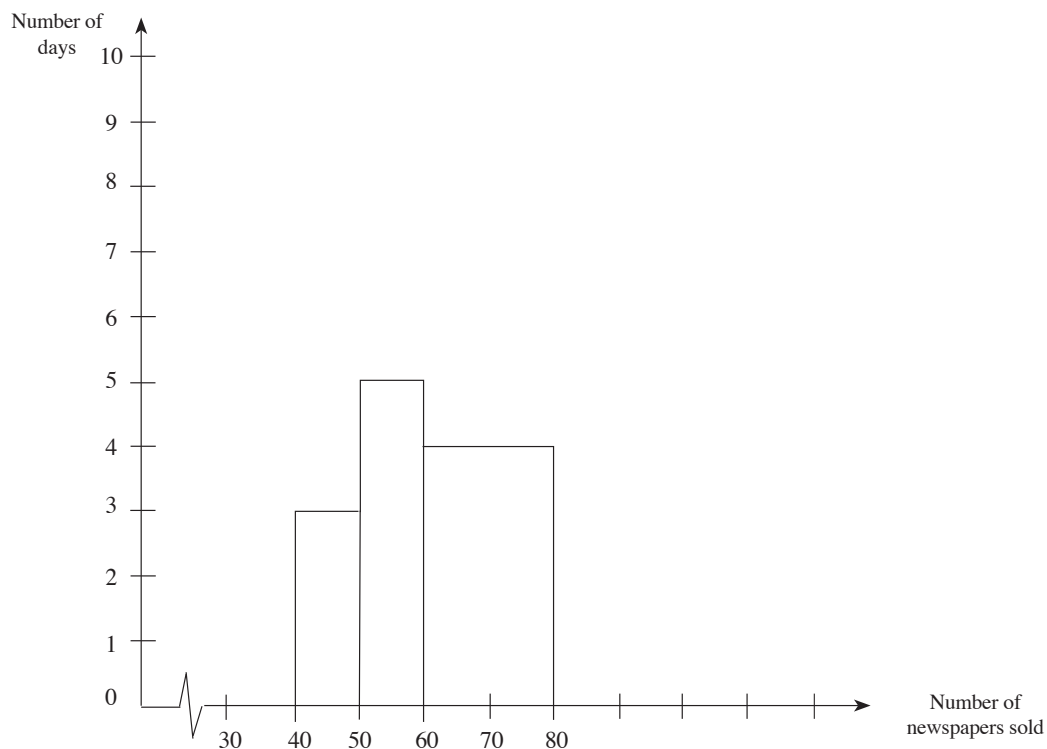
- (i) How much is $n(A \cap C)$?



- (ii) Express the shaded region in the Venn diagram using set notation.

5. An incomplete table and an incomplete histogram prepared with the information on the number of newspapers sold each day at a newspaper stall during the month of August are given below.

| | | | | | |
|---------------------------|---------|---------|---------|----------|-----------|
| Number of newspapers sold | 40 – 50 | 50 – 60 | 60 – 80 | 80 – 110 | 110 – 120 |
| Number of days | 3 | 5 | ... | 9 | ... |



- (i) What is the minimum number of newspapers that may have been sold on a day during this month?
- (ii) Fill in the blanks in the table.
- (iii) Complete the remaining part of the histogram.
- (iv) Construct the frequency polygon using the histogram.
- (v) The newspaper seller states that the number of days on which less than 80 newspapers were sold exceeds 50% of the total number of days in the month. Do you agree with this? Explain your answer.

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Mathematics II

Two Hours and Thirty Minutes

- * Answer **ten** questions by selecting **five** questions from **part A** and **five** questions from **part B**.
- * Each question carries 10 marks.
- * The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$, the volume of a right circular cylinder of base radius r and height h is $\pi r^2 h$, and the volume of a right circular cone of base radius r and height h is $\frac{1}{3}\pi r^2 h$.

Part A

Answer **five** questions only.

1. (a)

Pay Rs. 30 000 and take home a motorcycle worth Rs. 150 000.
The balance in 24 monthly installments of Rs. 5625.



- In purchasing the motorcycle, after the initial payment of Rs. 30 000, what is the remaining loan amount?
 - What is the monthly loan amount?
 - If the motorcycle is bought under the hire purchase scheme, find the extra amount that has to be paid.
 - Calculate the annual interest rate charged under this scheme.
 - A discount of 12% is given when an outright payment is made. Then what is the selling price of the motorcycle?
- (b) If a person invested Rs. 150 000 in a company which pays an annual dividend of 20% to buy 5000 shares of value Rs. 25 per share, find the annual dividend income he earns.
2. An incomplete table of values prepared to draw the graph of the function $y = (1 - x)(3 + x)$ is given below.

| | | | | | | | |
|-----|----|----|-----|----|---|---|----|
| x | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
| y | -5 | 0 | ... | 4 | 3 | 0 | -5 |

- Write down the value of y when $x = -2$.
 - Draw the graph of the above function by taking 10 small divisions along the x axis and the y axis to represent one unit as scale.
 - Using the graph,
 - write down the coordinates of the turning point of the function.
 - find the two values of x for which the value of the function is -2 .
 - what is the interval of values of x for which the function is positive?
 - write the function in the form, $y = b - (x + a)^2$.
3. (a) Find the matrix \mathbf{A} such that $\begin{pmatrix} 3 & 4 \\ -2 & 7 \end{pmatrix} - 2\mathbf{A} = \mathbf{I}$. Here \mathbf{I} is the identity matrix of order 2×2 .
- (b) Determine the factors :
- $2x^2 - 18$
 - $9a(x + 2y) - 5bx - 10by$

(c) Three times the number of students in the Advanced Level Science Stream of a certain school is one less than four times the number of students in the Advanced Level Arts Stream. If four students from the Science stream shift to the Arts Stream, then the number of students in both streams will be equal.

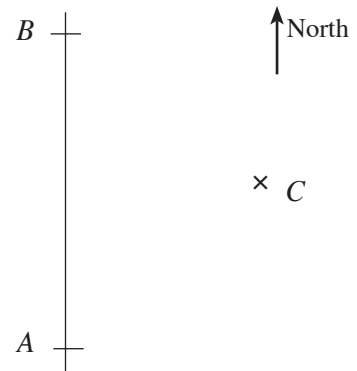
- (i) Construct a pair of simultaneous equations by taking the number of students in the Science Stream to be x and the number of students in the Arts Stream to be y .
- (ii) Solve the pair of equations.

4. (a) It was initially suggested to collect an equal amount of money from each student in a certain class, to buy an electric fan priced at Rs. 2400. However, it was later decided not to collect money from 5 students who had financial difficulties. Due to this, each of the other students had to pay Rs. 16 more than the amount that was initially suggested. Let x be the number of students in this class.

- (i) Express in terms of x , the amount that was initially suggested for collection from each student.
- (ii) Express in terms of x , the amount that the remaining students had to pay due to the decision not to collect money from five students.
- (iii) Construct a quadratic equation in terms of x with the above information, and by solving it, find the number of students there are in the class.

(b) Solve the quadratic equation $x^2 - 4x - 16 = 0$ by completing the square or by another method. (Take $\sqrt{5} = 2.24$)

5. (a) The tip C of a reef in a lake is seen to a point A at the same horizontal level on a straight bund of the lake which is pointed towards North from South, at a bearing of $038^\circ 20'$ and a distance of 150 m.



- (i) Copy the given figure onto your answer sheet and mark the above data on it.
- (ii) Using trigonometric ratios, calculate to the nearest metre, the shortest distance CP from C to the bund.
- (iii) What is the bearing of C from point B which is 80 m North of P and at the same horizontal level?

(b) A map has been drawn to the scale 1 : 25 000.

- (i) Find the actual distance in kilometres between two cities which are marked 10 cm apart from each other on the map.
- (ii) By what length is a distance of 3 km denoted on the map?

6. Information on the number of loaves of bread that was sold by a certain bakery on 30 days is given in the table :

| | | | | | | |
|--------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| Number of loaves of bread sold | 100 - 120 | 120 - 140 | 140 - 160 | 160 - 180 | 180 - 200 | 200 - 220 |
| Number of days (Frequency) | 2 | 4 | 8 | 12 | 3 | 1 |

- (i) To which class interval does the amount of bread sold on the most number of days belong?
- (ii) Find the mean number of loaves of bread sold during a day.
- (iii) Estimate the income received by the bakery during a month through the sale of bread, if a loaf of bread is priced at Rs. 60.
- (iv) The owner of the bakery claims that the maximum number of loaves of bread sold during a month by the bakery exceeds 5000 loaves. Could this statement be true? Give reasons for your answer.

Part B

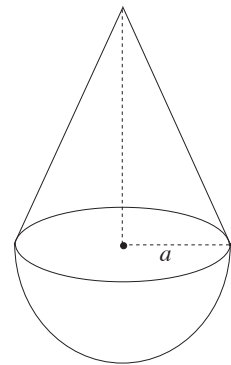
Answer **five** questions only.

7. (a) Cakes of soap have been displayed on a rack in a pattern, such that the bottom layer has 28 cakes, the next layer has 25 cakes and the one above that has 22 cakes.
- (i) Write down the first three terms of the number sequence according to which the cakes of soap have been placed on the rack, and name what type of progression it is.
 - (ii) According to this progression, how many cakes of soap are in the seventh layer?
 - (iii) If the top layer consists of only one cake of soap, how many such layers are there?
 - (iv) If the rack has 5 displays of soap placed in the above manner, determine whether the statement "The total number of cakes of soap in the 5 displays does not exceed 720" is true or false and justify your answer.

(b) Find the geometric mean of $2x^2$ and $18x^4$.

8. Using only a straight edge with a cm/mm scale and a pair of compasses, and showing the construction lines clearly,
- (i) construct the trapezium with $AB = 5.5$ cm, $\hat{BAD} = 60^\circ$, $AD = 4.6$ cm, $DC = 7.0$ cm and $AB \parallel DC$.
 - (ii) construct the locus of the point moving equi-distant from the two vertices B and C .
 - (iii) construct the circle with centre on the above locus and which touches DC at C .
 - (iv) Measure and write down the radius of the circle.
 - (v) Construct the tangent DE to the circle from D which is distinct from DC .

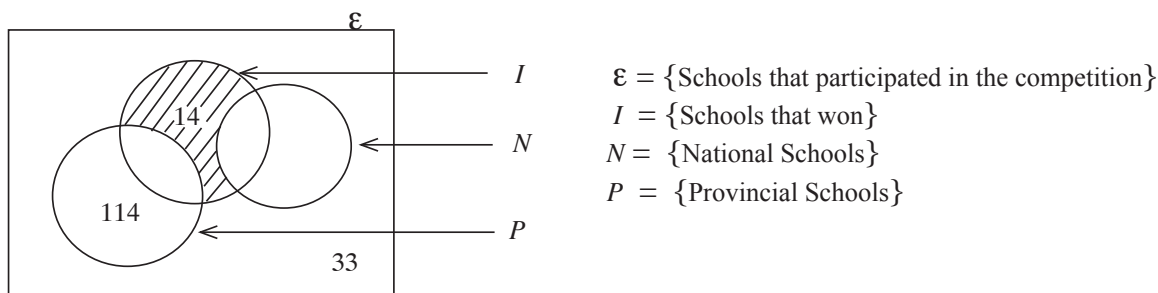
9. (a) The compound solid in the figure constructed with metal, consists of a solid hemisphere of radius a and a right circular solid cone of height three times the base radius.
- (i) Write down the height of the compound solid in terms of a .
 - (ii) Show that the volume of the compound solid is $\frac{5}{3} \pi a^3$.
 - (iii) This solid is melted and cylindrical rods of radius $\frac{1}{3} a$ and length $5a$ are made. Find the number of rods that can be made without wastage.



(b) Find the value using the logarithms table :

$$\frac{43.27^2}{\sqrt{0.0754 \times 852}}$$

10. National schools, Provincial schools and Private schools participated in a national competition. An incomplete Venn diagram containing information on the schools that participated is given below.



- How many Provincial schools **did not** win?
175 National schools and 130 Provincial schools participated in the competition. The number of National schools that **did not** win is 135.
- Copy the Venn diagram onto your answer sheet and using the above information, fill in the regions which are blank, with the relevant numbers.
- How many National schools won?
- Describe in words the region in the Venn diagram that shows 33 schools.
- Express the shaded region in the Venn diagram, using set notation.
- What is the probability of a school which is selected at random from those that participated in the competition, being a school that won?

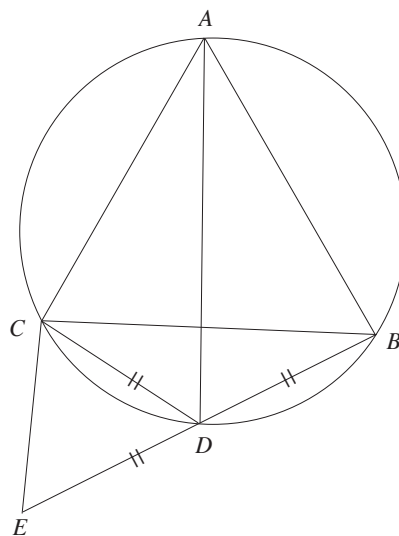
11. ABC is an equilateral triangle. D is a point on the circle such that $CD = DB$. The straight line BD has been produced to E such that $BD = DE$.

- (a) Providing reasons, find the magnitudes of the following angles.

- \hat{CDB}
- \hat{BCD}

- (b) Prove the following :

- AD is a diameter.
- CDE is an equilateral triangle.
- $CE \parallel AD$.
- $AD \perp BC$.



12. $\hat{ABC} = 90^\circ$ in the right triangle ABC . The straight line drawn parallel to BA , through X , the mid point of BC , meets the side AC at Y . The side BA has been produced to D such that $BA = AD$. The straight line drawn parallel to AC through the point D and the straight line XY produced, meet at Z .

- Draw a sketch with the above information marked on it.
- Give reasons why the quadrilateral $ADZY$ is a parallelogram.
- What is the relationship between the length AY and the length YC ?
Write down the theorem that you can use to provide the answer.
- The trapezium $DZYB$ is said to be isosceles. For this to be true, prove that $DZ = YB$.
- Show that the area of the trapezium $DZYB$ is six times the area of triangle XYC .